

RESEARCH ARTICLE

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Analysis of Mathematical Errors Committed by Grade Six Children with Mathematics Difficulties: Implications for Classroom Instruction

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Abstract: A growing body of research has shown that children with mathematics difficulties (MD) encounter problems in a range of mathematical tasks including mathematical computations, mathematical concepts and word problems. However, limited work has been accomplished to date that documented the children's specific difficulties or problems in each category of mathematical tasks. The present study examined whether or not children with MD face more difficulties with some operations in each category than other operations. The study selected 13 grade six children with MD from two primary schools in Addis Ababa, Ethiopia. Examination of the students' performance on a 50-item curriculum-based mathematics test showed that (i) on the computation subtest, the children performed significantly poorly on items with multiplication, division and mixed operations as compared to computation items that require addition and subtraction; (ii) on mathematical concepts, the children performed significantly worse on all items but they performed slightly better on principles and rules; and (iii) on word problems, the children performed significantly poorly on all five types of items (addition, subtraction, multiplication, division and mixed operations). Overall, the findings show that children with MD face difficulties with several types of mathematical (computation, concepts and word problem) tasks except computation items that require the application of addition and subtraction.

Keywords: children with mathematics difficulties, mathematical errors, mathematical concepts, computations, word problems

INTRODUCTION

A growing body of research has documented that children with mathematics difficulties (MD) experience problems in mathematics more frequently than their peers without MD (see Miller & Mercer, 1997). Studies have documented, for example, that children with MD encounter problems in a range of mathematical tasks including mathematical computations and word problems (Miller & Mercer, 1997; Zentall & Ferkis, 1993). Research has also shown that these children not only have lower scores on mathematics tests in comparison to age- or grade-matched typically achieving peers but function at a lower level than younger children without MD (Cawley & Miller, 1989; Fleischner et al., 1982).

In Ethiopia, like elsewhere in the world, many students consider mathematics as a difficult subject to learn. As a result, a considerable number of students hold less favorable attitude toward the learning of mathematics (Zeleke, 2001). While such conditions alone impact the students' achievement detrimentally, the school system exerts little effort, if any, to support the children. What is more, the school system does not officially recognize learning

difficulties and neither diagnosis/identification nor support services are available for children with learning difficulties including those with MD. The children attend general education classes and in the absence of support services, it would not be difficult to imagine how these children are at disadvantages compared to their typically achieving peers.

The absence of routine assessment and diagnosis of learning difficulties in Ethiopian primary schools forces local researchers to begin their research from identification of the children with MD. Given the limited resources available for such research in the country, studying these children has remained a difficult and demanding undertaking. As a result, only a few studies have been conducted on these children to date. The few available studies that investigated the characteristics of primary school children with MD (Grades 4-6) in Addis Ababa, Ethiopia (Zelege, 2004) have reported the following results.

- Children with MD scored substantially lower than their average achieving peers on all three mathematics subtests (computations, concepts and word problems) administered.
- Children with MD displayed a relatively lower level of performance on the concepts subtest than on either the computation subtest or word problem subtest.
- Children with MD performed relatively better on the computation subtest than on either the concepts subtest or the word problem subtest.

In general, the results were consistent with findings of several studies conducted in other countries (e.g., Jordan & Montani, 1997; Ostad, 1998; see also Montani, 2003). In view of the results pertaining to the mathematical concepts subtest, however, the mentioned studies provided a new insight in that children with MD not only have difficulties with word problems and computations, but also lack the conceptual knowledge necessary to solve the problems.

In light of classroom instruction, the above results may not show the big picture about the children's problems. If teachers have to use the results as a basis to support the children with MD in class, they need to know, in specific terms, about the children's problems. For example, although the results indicated that the children performed relatively better on the computations subtest, does this mean that the children are equally proficient on all kinds of computation items (e.g., items involving addition, subtraction, multiplication, division, and mixed operations)? Similar questions may be raised regarding the concepts subtest and the word problems subtest. The answers we obtain to such specific questions through research would be more useful from the perspective of classroom instruction than the results of the examination of the children's relative performance on the three subtests.

Accordingly, the items in each subtest were classified into more homogeneous sets of items and the errors committed by the children on these sets of items were examined (e.g., the computation subtest was divided into five more homogeneous sets of computation items involving addition, subtraction, multiplication, division, and mixed operations). Following this line of inquiry, the present study sought to answer the following questions.

- Do children with MD perform equally well on computation items involving addition, subtraction, multiplication, division, and mixed operations?
- Are the children equally proficient on different types of mathematical concepts items?
- Do children with MD perform equally well on word problem items involving addition, subtraction, multiplication, division, and mixed operations?

As mentioned above, the answers to these questions would provide valuable information for teachers and parents, among others, regarding the nature and specific areas of the children's difficulties. The results would also help educators and teachers to clearly understand the children's problems and the corresponding instructional interventions that are useful in supporting the children in class.

METHOD

The data analyzed in the present study is part of a relatively comprehensive data set that was collected to investigate the relationship between self-concept and mathematics achievement of children with mathematics difficulties and their classmates without mathematics difficulties. The original data set included various data

gathered from children who were then attending grades four through six in two government primary schools in Addis Ababa, Ethiopia.

Participants

The participants of this study were 13 grade six children with MD (8 girls and 5 boys). At the time of data collection, the children's mean age was 13.77 years (standard deviation [SD] = 1.69). Selected from an initial pool of 188 grade six children, the children with MD displayed consistent mathematics difficulties on a curriculum-based mathematics test which was administered to the children twice with a six-month interval. More specifically, the children with MD met two criteria:

- (1) scoring at or below the 25th percentile on a mathematics test at both times of measurement and
- (2) scoring in the average range on the Test of Nonverbal Intelligence – 3rd edition (TONI-3) (Brown et al., 1997).

Both criteria are frequently used in research with children who have mathematics difficulties or learning difficulties (e.g., Badian, 1999; Siegel & Ryan, 1989; see also Geary et al., 2000).

Data Collection Tools

Data were collected using curriculum-based mathematics test, Test of Nonverbal Intelligence (TONI-3) and teacher ratings. The mathematics test was developed in such a way that it closely matched the mathematics curriculum for grade six. It can thus be considered as having better content validity for the present sample than commercially available standardized tests. With a total of 64 items, the original test was composed of three subtests: computations (24 items), concepts (24 items) and word problems (16 items). For the purpose of the present study, the three subtests were used but these included only 50 items. The 14 items which the author could not classify in one of the new smaller categories of items were excluded from the analysis. The internal consistency of the original test items as estimated by Cronbach's alpha was .91. Also, the test-retest reliability coefficient with a six-month interval was .84. The data analyzed in the present study were obtained from the first administration of the mathematics test.

Besides, TONI-3 was used to estimate the children's nonverbal IQ. Although a test which measures both verbal and nonverbal IQ was desirable, such a test was not available for use with Ethiopian children. As a nonverbal IQ test, TONI-3 requires each participant to complete 45 nonverbal matrices, one at a time. The authors (Brown et al., 1997) have reported satisfactory coefficients of internal consistency (above .92) and test-retest reliability (above .90) as well as evidence of validity for the test. In order to familiarize the children with the task, five practice items were administered before the 45 main items. The children scored in the average range (Mean = 84.46, SD = 8.37) on TONI-3.

Furthermore, at the beginning of data collection, teachers rated whether each child in the initial sample had reading difficulties, visual impairment, hearing impairment or speech problems on a two-point (yes or no) scale developed for the study. The 13 children with MD who participated in the present study were rated by their teachers to have none of the problems mentioned above. That is, the children had no difficulty other than MD. The teachers had ample time and opportunity (nine months of teaching) to observe the children.

Data Analysis Procedure

In analyzing the data obtained from the 13 children with MD, first the items in each subtest were classified in more homogeneous sets. The children's levels of performance or the errors they committed on these sets of items within each subtest were then compared to determine the children's specific areas of difficulties. In evaluating the children's performance on each subtest or homogeneous sets of items, 50% correct was used as a cut-off score to distinguish satisfactory and poor performance following the guideline set by the Education and Training Policy of the country (Transitional Government of Ethiopia, 1994). One-sample t-test and paired-samples t-test were used to test whether the children's performance on each set of items was significantly different from the cut-off score or from their performance on the other sets of items, respectively.

Table 1. Means and Standard Deviations of Correct Answers by Subtest

Mathematics Subtest	Number of Items	Mean	Standard Deviation
Mathematics Computation	20	7.00	2.00
Mathematics Concepts	15	2.77	1.42
Word Problems	15	3.54	1.05
Total Test	50	13.31	2.78

Table 2. Means and Standard Deviations of Scores on Computation Items

Computation Items by Operation	Number of Items	Mean	Standard Deviation
Computation: Addition	4	3.00	0.71
Computation: Subtraction	4	2.08	1.12
Computation: Multiplication	4	0.62	0.77
Computation: Division	4	0.38	0.51
Computation: Mixed Operations	4	0.92	0.76

RESULTS

First, the children's scores on the three subtests are examined. The scores of the 13 children on each subtest as well as on the total test are presented in **Table 1**. One can easily observe from the data in **Table 1** that the children's performance was generally poor relative to the 50% correct cut-off score.

The data indicate that whereas the mean percentage correct score on the computation subtest was 35 (that is, $7/20 \times 100$), the corresponding means for the concepts and the word problems subtests were 18.47 and 23.60, respectively. One can therefore say that despite their poor performance on all the three subtests, the children performed relatively better on the computation subtest. In contrast, the children committed more errors on the concepts subtest.

Admittedly, the above analysis based on subtest scores provides general information on the performance of the children. However, as argued earlier, more useful information particularly for classroom instruction could be obtained by classifying the items in each subtest into more homogeneous categories to see the difficulties of the children more clearly. The analysis presented below is done taking this argument into account.

Performance on Computation Items

The items in the computation subtest were classified into five more homogeneous categories comprising the four arithmetic operations as well as mixed operations where two or more operations are involved. The resulting scores are shown in **Table 2**. The mean scores indicate that the children's performance on the computation items involving addition and subtraction was each above average. Particularly on items involving addition, the children's mean percentage score was 75, which is significantly higher than the criterion score (50% correct) for satisfactory performance ($t = 5.10$, $df = 12$, $p < .001$). This means that even though the children have mathematics difficulties, they perform well above average on the computation items involving addition.

The children's performance on subtraction items was also satisfactory. However, unlike the mean score for the addition items, the mean score (52% correct) on subtraction items was not significantly larger than the criterion score ($t = 0.25$, $df = 12$, $p > .05$). Thus, the children had average performance on subtraction items.

In direct contrast, the mean scores on those computation items involving multiplication ($t = -6.50$, $df = 12$, $p < .001$), division ($t = -11.50$, $df = 12$, $p < .001$), and mixed operations ($t = -5.11$, $df = 12$, $p < .001$) were far below average or significantly lower than the criterion score. Put differently, the children committed a large number of errors when the computation items involved multiplication, division, or mixed operations.

The use of paired-samples t -test similarly showed that the children had significantly better performance on computation items involving addition than each of the other four categories of computation items ($p < .01$ in each case). Also, the children had significantly larger mean score on computation items involving subtraction than those involving multiplication, division, or mixed operations ($p < .01$ in each case). In contrast, the mean scores on multiplication items, division items, and items that involved mixed operations were not significantly different from each other.

Table 3. Means and Standard Deviations of Scores on Concepts Items

Classification of Concepts Items	Number of Items	Mean	Standard Deviation
Concepts: Fractions/Decimals	5	0.31	0.48
Concepts: Principles/Rules	5	1.62	0.77
Concepts: Units/Measurement	5	0.85	1.14

Table 4. Means and Standard Deviations of Word Problem Scores

Word Problem Items by Operation	Number of Items	Mean	Standard Deviation
Word Problem: Addition	3	0.92	0.64
Word Problem: Subtraction	3	0.69	0.63
Word Problem: Multiplication	3	0.54	0.66
Word Problem: Division	3	0.69	0.63
Word Problem: Mixed Operations	3	0.69	0.63

Performance on Items that Tap Mathematical Concepts

The items included in the mathematical concepts subtest were classified into three categories each containing five items: concepts of fractions or decimals, concepts involving principles or rules and concepts pertaining to units/measurement. The mean scores for the three categories are shown in **Table 3**. The data clearly show that the children's performance on all three categories of mathematics conceptual items was far below the 50% correct cut-off score. That is, compared to the 50% correct cut-off score (that is, a score of 2.5 out of 5), the children had significantly lower mean score on fraction/decimal items ($t = -16.45$, $df = 12$, $p < .001$), on items that involved rules/principles ($t = -4.15$, $df = 12$, $p < .01$), and on items that involved units/measurement ($t = -5.22$, $df = 12$, $p < .001$).

Thus, unlike their performance on computation items, the children performed poorly (that is, far below the cut-off score) on all three categories of conceptual items. However, paired-samples t-test showed that the children's mean score on conceptual items involving principles or rules was significantly better than their mean score on items involving fractions or decimals ($t = 4.98$, $df = 12$, $p < .001$) but not on measurement items ($t = 1.87$, $df = 12$, $p > .05$). Also, the children's mean scores on fraction items and measurement items were not significantly different ($t = -1.72$, $df = 12$, $p > .05$).

Performance on Word Problem Items

Like the computation subtest, the items of the word problem subtest were classified into five categories involving the four arithmetic operations and mixed operations. The resulting mean scores are presented in **Table 4**. Compared to the 50% correct cut-off score (that is, a score of 1.5 out of 3), the children's mean scores for all five categories were significantly lower ($p < .01$ in each case).

A further analysis of the data using paired-samples t-test showed that the mean scores did not show any statistically significant difference when compared pair-wise ($p > .05$ in each case). Overall, the children's mean scores on all five categories of word problem items were more or less comparable but significantly lower than the criterion score.

DISCUSSION

The importance of error analysis as an effective method in providing more detailed information about learners' errors has been well documented (see Kingsdorf & Krawec, 2014). More importantly, error analysis provides important information for teaching as well as intervention that aims to improve children's performance on a number of mathematical areas. Further, several researchers (e.g., Herholdt & Sapire, 2014) point out that error analysis provides a learning opportunity to teachers to improve their mathematics teaching and this holds not only for children with MD but also for the general student population.

The purpose of this study was to examine the errors committed by 13 sixth grade children with MD on a mathematics test comprising three subtests: computations, concepts and word problems. In an effort to obtain a clearer picture of the children's errors, each subtest was further divided into more homogeneous sets of items.

Overall, the results indicated that the children's performance on the test was substantially lower than the 50% correct cut-off score showing that the children committed a large number of errors. More specifically and in response to the research questions, the study yielded the following results. The children did not perform equally well on the five categories of the computation subtest. They performed well above and slightly above the 50% correct cut-off score on computation items involving addition and subtraction, respectively. But they scored significantly below the cut-off score on the remaining three categories (multiplication, division and mixed operations).

- The children performed substantially below the cut-off score on all three categories of the mathematical concepts items. However, they performed relatively better on concepts involving principles/rules and relatively poorly on fraction/decimal concepts. The children performed consistently and significantly below the cut-off score on all five categories of word problem items (addition, subtraction, multiplication, division and mixed operations).
- The children performed substantially below the cut-off score on all three categories of the mathematical concepts items. However, they performed relatively better on concepts involving principles/rules and relatively poorly on fraction/decimal concepts.
- The children performed consistently and significantly below the cut-off score on all five categories of word problem items (addition, subtraction, multiplication, division and mixed operations).

Previous research generally indicated that children with MD perform better on computation items than on other mathematics subtests notably word problems (e.g., Jordan & Montani, 1997; Ostad, 1998). However, while such a conclusion is generally true, it is misleading because the children's performance was not equally good on all types of computation items. Thus, to draw more accurate conclusion, the findings of this study underscored the importance of examining the children's performance on tests that focus on more homogeneous or specific mathematics tasks (that is, by going beyond the often used general categories such as computations and word problems).

Overall, the children performed satisfactorily on two categories of the computation subtest, namely computation items involving addition (mean = 75% correct) and subtraction (mean = 52% correct). In direct contrast, by far the worst level of performance the children displayed was in mathematical concepts that involved fractions/decimals. While this result is in agreement with findings of some previous research (see Montani, 2003), it suggests that problems pertaining to mathematical concepts are even more difficult than word problems. That is, even though word problems are generally considered more difficult than computations, according to the results of this study, items on mathematical concepts involving fractions or decimals are even more difficult for children with MD. Previous research has also shown, with at-risk (for MD) children (e.g., Malone & Fuchs, 2017) and with children who have MD (Hecht & Vagi, 2010), that problems involving fractions present persistent and significant difficulty for elementary school children.

CONCLUSION

The above findings show that the children with MD performed significantly lower than the 50% correct criterion on all mathematical tasks except computation items that require the application of addition and subtraction. Even addition and subtraction were found to be difficult for the children when the items are presented in the form of word problems suggesting that the children have problems converting the word problems into equations which can lead them to the solutions. The children's performance was the lowest on items of mathematical concepts pertaining to decimals/fractions.

Implications for Classroom Instruction

The results of this study have several implications for classroom instruction. The main implications are described below.

1. Mathematics has been taught for a long time as rote application of basic skills without giving attention to strategy instruction. Intervention research focusing on mathematical problem solving has provided evidence that children who lack problem solving strategies need explicit instruction to facilitate, among

others, their understanding, executing and evaluation of the problems (see Ostad, 1999). Thus, children need to be taught what strategies are available to solve one kind of mathematical problem or another and how to use the strategies. Besides, to minimize difficulties the children may face as well as to give life to the problems, selecting concrete tasks from the children's daily life experiences would be of considerable help.

2. The results of this study indicated that the children with MD displayed the poorest level of performance on the mathematics concepts subtest. Experience and observations indicate that in Ethiopian primary schools, mathematics teaching tends to focus more on rote memorization than understanding, on knowledge of computational procedures than on knowledge of concepts and problem solving skills. The results also seem to highlight this trend. On the other hand, there is evidence to support that children who better understand concepts also tended to solve problems better in several mathematical domains (Rittle-Johnson & Siegler, 1998) suggesting that the focus in teaching mathematics should center on mathematical concepts. It would seem logical to assume that if children have consolidated knowledge of mathematical concepts, then they would encounter difficulties relatively rarely. Accordingly, teachers of children with MD should exert every possible effort to provide explicit instruction on mathematical concepts.
3. Comparison of the children's mean scores on the computation and word problem subtests indicate that even though the same operations were involved in both subtests, the children performed relatively better on the computations subtest. This implies that word problems present additional difficulties to the children and perhaps this has something to do with the children's difficulty in understanding the problem and the procedures necessary to convert the word problem to a mathematical equation with an unknown (or variable). If this is a valid assumption, then teaching should focus on encouraging children to follow some specific procedural steps to solve the problem accurately. First teaching the procedures and then encouraging the children to exercise them by themselves in solving word problems and finally providing feedback on their work are important steps that should be taken to support the children in class.
4. The time that would be spent in supporting children with MD need to be used wisely taking into account the children's relative difficulties. That is, more time should be allotted for difficult problems such as multiplication and division rather than addition and subtraction. Similarly, more time should be spent in teaching fractions/decimals and how problems involving fractions are solved than teaching mathematical rules/principles.

Limitation of the Study

One limitation of the present study is the small number of children ($n = 13$) with MD who participated in the study. Although children with MD are generally small in number compared to typically achieving children, readers should generalize the results cautiously for the sample size is very small to detect all mathematical errors that children with MD could commit.

A second limitation of the study arises from its cross-sectional nature. The children with MD who participated in this study were also from one grade level (Grade 6) only. For these two reasons, it is difficult to ascertain whether the patterns of mathematical errors observed in this study have emerged in earlier grades or typically characterize children with MD at the sixth grade level.

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Please cite: Teketel, S. Z. (2022). Analysis of Mathematical Errors Committed by Grade Six Children with Mathematics Difficulties: Implications for Classroom Instruction. *Journal of Research in Didactical Sciences*, 1(1), 12500. <https://doi.org/10.51853/jorids/12500>

Received: 07 February 2022 • Accepted: 28 August 2022